

Semester One Examination, 2023

Question/Answer booklet

**MATHEMATICS
SPECIALIST
UNIT 3**

**Section One:
Calculator-free**

If required by your examination administrator, please place your student identification label in this box

WA student number:

In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	5	5	50	52	35
Section Two: Calculator-assumed	9	9	100	94	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section One: Calculator-free

35% (52 Marks)

This section has **five** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(8 marks)

Two lines, L_1 and L_2 are defined as follows:

$$L_1: \begin{cases} x = 4 + 4t \\ y = 4 + t \\ z = 6 + t \end{cases} \quad L_2: r = \langle -8, 11, -3 \rangle + s\langle -2, -3, 1 \rangle$$

- (a) Show that the two lines intersect, then determine the coordinates of the point of intersection.

(5 marks)

- (b) Find the Cartesian equation of the plane that contains these two lines.

(3 marks)

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Question 2

(11 marks)

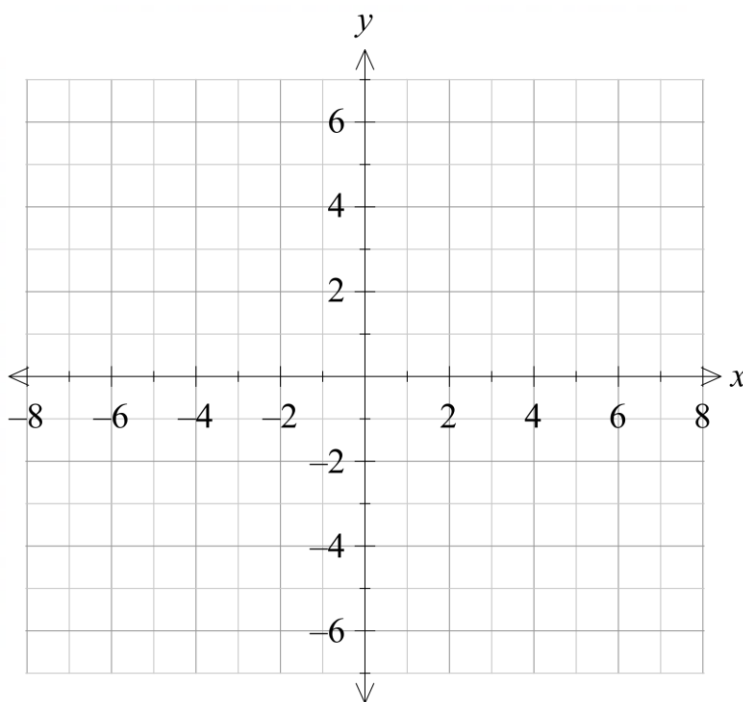
Consider the graph of the following function

$$f(x) = \frac{x^2 + 2x + 5}{x + 1}$$

(a) Determine $f'(x)$. (2 marks)

(b) Determine the coordinates of any stationary points. (2 marks)

(c) Sketch the graph of $f(x)$ on the axes below, indicating all intercepts, asymptotes and stationary points. (6 marks)



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DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

- (d) If the graph of $g(x) = |x - a|$ is drawn on the axes in part (c), determine the value(s) of a , ($a < 0$) such that the graphs of $f(x)$ and $g(x)$ have one point of intersection.

(1 mark)

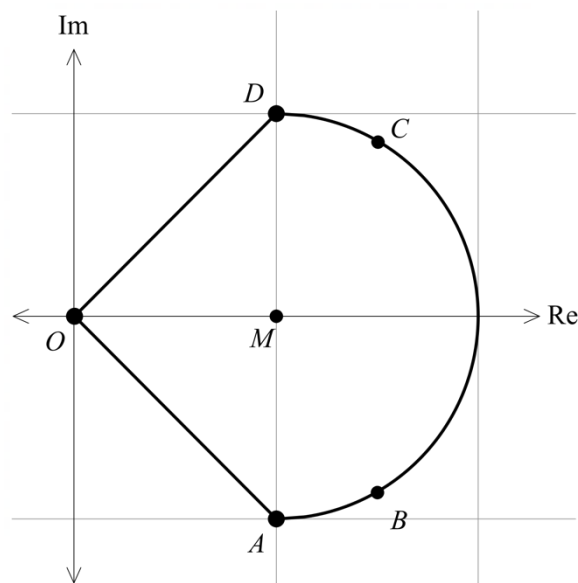
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Question 3

(9 marks)

The Argand diagram below shows a right-angled triangle AOD , with semicircle $ABCD$ centred at M .



- (a) Given A represents the complex number $1 - i$, determine the complex number representing D . (1 mark)

- (b) State the locus of points that define semicircle $ABCD$. (3 marks)

The rays MC and MB form angles of $\frac{\pi}{3}$ with the positive direction of the real axis.

Let C be the complex number z_C , and B be the complex number z_B .

(c) Determine, in polar form, z_C . (3 marks)

(d) Explain why $\overline{z_C} = z_C \operatorname{cis}\left(-\frac{2\pi}{3}\right)$. (2 marks)

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Question 4

(17 marks)

The polynomial $x^3 + ax^2 + bx + c$ has the following features:

- a factor of $(x - 1)$
- a remainder of -8 when divided by $(x + 1)$ and
- a remainder of 19 when divided by $(x - 2)$.

(a) Use the above information to show that $a = 3$, $b = 3$ and $c = -7$.

(5 marks)

(b) (i) Explain why $x^3 + 3x^2 + 3x - 7 = 0$ cannot have one complex root.

(1 mark)

(ii) Using parts (a) and (b)(i), solve $x^3 + 3x^2 + 3x - 7 = 0$.

(3 marks)

- (c) Solve $z^3 = 1$, giving your solutions in Cartesian form. (3 marks)

The solutions to $z^3 = 1$, can be transformed to the solutions of $x^3 + 3x^2 + 3x - 7 = 0$, using the transformation $x = az + b$, where a and b are positive real numbers.

- (d) (i) Determine the values of a and b , in the transformation $x = az + b$. (3 marks)

- (ii) Hence, deduce the type of triangle formed by the solutions $x^3 + 3x^2 + 3x - 7 = 0$. Justify your answer. (2 marks)

Question 5

(7 marks)

Three functions are defined as follows:

$$\begin{aligned}f(x) &= x^2 - 4 \\g(x) &= \sqrt{x} \\h(x) &= \frac{2x - 4}{x + k}\end{aligned}$$

(a) (i) Determine an expression for $(f \circ g)(x)$. (1 mark)

(ii) Determine the range of $(f \circ g)(x)$. (1 mark)

(b) Determine the domain for the function $y = g(f(x))$. Justify your answer. (2 marks)

A self-inverse function is one for which $f(x) = f^{-1}(x)$, for all values of x in the domain.

(c) Determine the value of k such that $h(x)$ is a self-inverse function. (3 marks)

End of questions

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Supplementary page

Question number: _____

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